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Book Review: *Fractal Growth Phenomena*

Fractal Growth Phenomena, Tamás Vicsek, World Scientific, Singapore, 1989.

The past decade has witnessed an explosive activity in the study of disorderly growth phenomena. Much of the impetus behind these investigations has been the realization that random growth processes often lead to the formation of self-similar or self-affine fractal patterns. Significant progress has been achieved by studying simplified models with rich dynamical properties and finding scaling laws using various theoretical considerations. The most important aspect of these developments is that through fractal concepts it has recently become possible to describe a variety of far-from-equilibrium phenomena, which, despite their importance, until the last decade had not been investigated intensively because they were viewed as intractable.

The range of phenomena in which fractal patterns and temporal scaling have been observed in experiments and in simulations is overwhelmingly wide and varied. The examples for such patterns range from the shape of polymers and gels to the structure of the lungs and the patterns of the heart beat. The field of colloidal science has been redefined by the discovery of the fractal structure and the dynamical evolution of diffusion-limited aggregation processes. Unstable growth in phenomena governed by the Laplace equation is now unified through a better understanding of fractally formed clusters in diffusion-limited aggregation and cluster formation. Our knowledge of the dynamics of fractal surfaces has also undergone major developments in recent years and promises to continue to be a fertile area for future research.

The present book under review is a welcome addition to the small, but growing, number of books on fractals. In particular, it is the first comprehensive book devoted entirely to the subject of fractal growth. It is written for a wide audience, including advanced graduate students and those who would like to learn about and do research in this field. Its aim is to provide a pedagogical overview of the subject that would make it useful for both learning the concepts and using them in one's research.

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This book meets an especially important need, because disorderly growth has been among the least understood areas of nonequilibrium statistical physics and fractal concepts represent the most promising approach to describing growth under far-from-equilibrium conditions. I recommend this book strongly to students entering the field of non-equilibrium statistical mechanics, where there are numerous and exciting open problems which could be treated by fractal and scaling concepts, and to researchers seeking a comprehensive review of a particular growth phenomenon. To nonspecialists, this book will also be quite useful as a general introduction to fractals, and growth phenomena in particular.

Tamás Vicsek is one of the leading pioneers of the study of fractal growth phenomena, having contributed to the subject widely and effectively. Thus, he is in an excellent position to write a technical and a pedagogical account of the most important advances in this field. One of the most novel aspects of the book is inclusion of many examples throughout. This is a particularly welcome addition, because other fractal books either do not give any examples or the examples are only algorithms for generating fractal patterns without understanding them. Vicsek's examples, on the other hand, are all carefully chosen to illustrate some of the concepts, like measures, multifractals, and self-affinity, which seem simple, but in reality can be quite elusive.

The book is divided into three parts: part I deals with various aspects of fractal geometry, from basic definitions of various types of fractals to technical descriptions of how fractal dimension can be measured. The second part consists of four chapters, each dealing with a different type of cluster growth phenomenon. The first class of cluster growth models are those in which the evolution of the interface is defined by simple local growth rules. These models often lead to complex fractal patterns and have been extensively used for describing disordered growth phenomena, like invasion percolation and other spreading processes.

The second chapter of part II is devoted to the diffusion-limited aggregation (DLA) model, introduced about 10 years ago by Thomas Witten and Leonard Sander. DLA is perhaps the most celebrated example of a fractal growth phenomenon. The great interest in the DLA model stems from the fact that, while it is one of the simplest of stochastic models to define, despite considerable effort, it has so far eluded any form of concrete mathematical description and remains one of the most exciting and challenging problems in physics. DLA is a model for a rich variety of physical, chemical, and biological phenomena that are governed by the Laplace equation. In contrast to the local growth models, due to the long-range nature of the Laplacian field and moving boundry condition, DLA is a nonlocal and nonlinear process. Vicsek discusses several physical realizations of the DLA model in part II and in part III of the book, which is devoted to fractal pattern formation. Many of the theoretical approaches for describing DLA and its variants are also reviewed and described.

Vicsek also discusses the dynamics of fractal surfaces, which is a topic that has received considerable attention in the past few years. Rough surfaces are self-affine fractals and are formed when the evolution of the interface is governed by a balance between random fluctuations and the surface tension which tends to smooth it. The interplay between these forces leads to the formation of marginally stable fronts that are globally flat, but have nonvanishing fluctuations on all length scales. As discussed by Vicsek, an important development in this particular area has been the convergence of the results of numerical and theoretical studies of the dynamic scaling of the fluctuating surfaces. More recently, experimental evidence has also been found for the dynamic scaling behavior of the interfaces in two-fluid flow experiments in porous media.

One of the most important applications of fractal and scaling concepts is in the description of the structure and dynamics in phenomena which can be treated in terms of the aggregation of clusters of particles. Vicsek discusses how the fractal dimension of the aggregates is determined from computer simulation models and from experiments on colloidal aggregates. The dynamics of aggregation processes is studied through the evolution of the cluster size distribution and Vicsek discusses the dynamic scaling description of several important classes of aggregation phenomena. The experimental verification of dynamic scaling represents one of the most important confirmations and successes of fractal and scaling concepts in fractal growth phenomena.

The last part of the book, part III, deals with fractal pattern formation and consists of two chapters, one on computer simulations of diffusion-limited pattern formation and one on the experimental studies of Laplacian growth phenomena. Vicsek discusses the models and techniques which are used to study diffusion-limited pattern formation, at a level that can be read both as an introduction and as a reference for further research. The chapter on experimental studies of Laplacian pattern formation provides one of the most comprehensive discussions of the experimental studies of fractal viscous fingering, and effects of anisotropy on the patterns.

Another novel aspect of the book lies in the appendices, in which Vicsek provides a simple computer algorithm and a description for building a Hele–Shaw cell for generating fractal patterns. Discovery in this field has been through many computer simulations and desktop experiments and the reader is urged to follow Vicsek's suggestions and discover a wealth of fractal patterns waiting to be described.

Despite the overall excellent quality of the book, there are a few problems that should have been avoided. I found many typographical errors in the book and the quality of the paper is such that often one can see through a page. Both of these problems should have been corrected by the publisher. Also, in order to make this a complete textbook on fractals, it would have been quite helpful if there were homework-type problems added at the end of each chapter. Some of the important developments in several areas, such as fracture, multifractals, biological pattern formation, and the dynamics of self-affine fractals, are not discussed. This is perhaps a problem inherent to any book written on a rapidly developing field. Finally, it would have been quite useful to have more detailed theoretical and mathematical treatments of some of the topics, but this could also be due to the fact that at the present time not many theories of fractal growth phenomena based on first principles exist.

In summary, Vicsek has succeeded in producing a classic book on growth phenomena. It is an extremely readable and useful book and I recommend it highly to anyone looking for a good place to start learning about fractals and growth phenomena in particular.

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